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The temperature and field dependence of the enhanced paramagnetic susceptibility due to spin fluctuations

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Abstract. The contribution of spin fluctuations to the temperature and field dependence of the low-temperature spin susceptibility $\chi(T)$ of exchange-enhanced paramagnets in finite magnetic fields is studied on the basis of the Fermi-liquid approach.

It is shown that the proper evaluation of the terms in the free energy that are dependent on magnetic field H results in the appearance of the logarithmic temperature contribution to $\chi(T)$, the existence of which was predicted earlier in Fermi-liquid approaches only qualitatively. Along with the logarithmic term, a new significant one which is proportional to $T^{5/2}/H^{1/2}$ has been found. The latter term can significantly change the usual interpretation of the experimental data on the magnetic susceptibility in terms of the power series over the temperature and magnetic field.

The reasons for these contributions not being found in previous considerations are discussed.

1. Introduction

The problem of the influence of spin fluctuations (SF) on the susceptibility of enhanced paramagnets has been discussed for many years, and a lot of results are available which provide a good understanding of some features of the observable temperature and field behaviour over wide temperature and magnetic field ranges. Nevertheless, a number of discrepancies between theories and experimental data as well as numerous contradictions between the results of different theoretical approaches have been the subject of much controversy.

The existence of detailed reviews of these problems (see, for example, [1–3]) permits us to restrict ourselves here to a brief enumeration of the main difficulties.

The theoretical efforts were directed basically towards the explanation of: (i) the abnormally strong temperature dependence of $\chi(T)$ for nearly ferromagnetic Fermi systems for $T \ll T_F$ (T_F is the Fermi temperature), and, especially, (ii) a non-monotonic temperature behaviour of $\chi(T)$ (in particular, the existence of a maximum in the temperature–susceptibility curve for Pd, U_2C_3, $CeMn_3$, etc). The confirmation of the key role of the incoherent SF in the low-temperature thermodynamics of such materials is the commonly recognized result of these efforts.

There are two groups of contradicting results concerning the temperature behaviour of $\chi(T)$. In a number of papers [5–8] it was shown that the SF give rise to 'doubly enhanced' T^2-terms in the temperature expansion of $\chi(T)$ (as compared with the 'singly enhanced' T^2-term in Stoner's theory):

$$\chi(T) = \chi(0)(1 - \alpha S^2 T^2) \qquad (1)$$

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(here and below the system of units $k_B = \mu_B = \hbar = 1$ is used, and all values are normalized to T_F), where S is the Stoner enhancement factor, and $\chi(0) = S\chi_P$, where χ_P is the Pauli susceptibility. According to (1), the characteristic temperature scale for changing $\chi(T)$ decreases from T_F to $T_{SF} = T_F/S \ll T_F$, and attempts were made to explain the observable susceptibility maximum either through the effects of the complicated band structure or through the presence of impurities and defects in samples [9]. The considerations were performed either in the Fermi-liquid approach [8], or by making use of particular microscopic models [5–7].

In a series of other works based either on the microscopic approach [1, 2], or on the general Fermi-liquid theory [4], it was claimed that in the temperature expansion of $\chi(T)$, along with the usual T^2-term, a logarithmic term has to be present:

$$\chi(T) = \chi(0)\left(1 - \gamma S^n T^2 \ln \frac{T}{T^*}\right) \tag{2}$$

where $n = 1$ [4], or $n = 3$ [1, 2].

Such a temperature dependence can explain the existence of a maximum in $\chi(T)$; however, the problem is still far from being resolved. In a number of works the conclusion as regards the appearance of logarithmic terms was shown to be erroneous, because when these terms are carefully collected the total elimination of various ones (the quasiparticle density of states, effective mass, vertex part etc) takes place [8, 10]. It was also indicated that the presence of such logarithmic terms must cause divergence of $\gamma(H)$ (the coefficient of the linear-in-T term in the specific heat) as $T \to 0$ [11]. Finally, the interrelation of these results with the conclusions of the first group of works [5–8] concerning the absence of logarithmic contributions is not obvious, and the ranges of applicability of these results are not clear.

In the present work, the Fermi-liquid approach is used to show that at finite magnetic fields the SF contribution to the free energy F is a complicated function of the parameter $\ell = 2B/T$ (where $2B$ is the energy of the spin splitting), and, therefore, the behaviours of the thermodynamic characteristics for different temperature regions ($\ell \ll 1$ and $\ell \geqslant 1$) are essentially different. That is, in weak but finite magnetic fields (when $\ell \geqslant 1$ and $T \gg H$) the new temperature contributions arise, and, for $\ell \gg 1$, these can be evaluated analytically. In the previous works, devoted to SF contributions to the thermodynamic characteristics of enhanced paramagnets, however, only the case of extremely weak magnetic fields ($\ell \ll 1$) was considered. We have found the 'singly enhanced' logarithmic term $T^2 \ln T$ at finite H, which for $H \to 0$ corresponds to the zero-field $ST^2 \ln S$-term of Beal-Monod *et al* [5]. In addition to this term, a new significant contribution to the spin susceptibility, which is proportional to $T^{5/2}/H^{1/2}$, has been obtained. The width of the temperature range of the existence of these new contributions is proportional to S—that is, this range may be large enough for the case of strongly enhanced paramagnets.

The technique for the extracting of the temperature contributions to the free energy described in the present paper can be used for the calculation of the whole spectrum of thermodynamic parameters of the enhanced paramagnets at finite magnetic fields.

2. The main equations

In the model of the Fermi liquid, when only the spin part of the interelectronic interaction approximated by single constant Ψ_0 is taken into account, the SF correction to the free energy $\delta F(M, T)$ (M is a magnetization) has the form [12]

$$\delta F = \frac{1}{2} V \sum_k \int_0^1 \frac{\mathrm{d}\lambda}{\lambda}\, \lambda \Psi_0 \langle \delta \boldsymbol{m}\, \delta \boldsymbol{m} \rangle_{\lambda k}. \tag{3}$$

Here $\langle \delta \boldsymbol{m}\, \delta \boldsymbol{m} \rangle_{\lambda k}$ is an averaged square of the spectral density of the SF (in which Ψ_0 is replaced by $\lambda\Psi_0$), and $k = (\boldsymbol{k}, \omega)$.

By making use of the fluctuation-dissipation theorem, and the expression for $\langle \delta \boldsymbol{m}\, \delta \boldsymbol{m} \rangle_{\lambda k}$ in the random-phase approximation, and after integrating over the coupling constant in (3), one can find

$$\frac{1}{V}\, \delta F = \frac{1}{2} \sum_{k,\omega} \coth\frac{\omega}{2T}\, \mathrm{Im}\Big(\ln(1 + \Psi_0 \chi_0^{zz}) - \Psi_0 \chi_0^{zz} + \sum_\sigma [\ln(1 + \Psi_0 \chi_0^{\sigma,-\sigma}) - \Psi_0 \chi_0^{\sigma,-\sigma}]\Big). \tag{4}$$

Here χ_0^{zz} and $\chi_0^{\sigma,-\sigma}$ are the longitudinal and transverse dynamical spin susceptibilities of quasiparticles in the absence of interaction, given by

$$\chi_0^{zz}(k, \omega) = \sum_{p,\sigma} \frac{n^\sigma_{p+k/2} - n^\sigma_{p-k/2}}{\omega - \varepsilon_{p+k/2} + \varepsilon_{p-k/2}}$$

$$\chi_0^{\sigma,-\sigma}(k, \omega) = 2 \sum_p \frac{n^\sigma_{p+k/2} - n^{-\sigma}_{p-k/2}}{\omega^\sigma - \varepsilon_{p+k/2} + \varepsilon_{p-k/2}}.$$

In the latter expression, the following notation is used: $\omega^\sigma = \omega - 2\sigma B$, where $\sigma = \pm 1$ is a spin quantum number; and $n_p^\sigma = n(\epsilon_{p\sigma})$ is a Fermi function for quasiparticles with the energy $\epsilon_{p\sigma} = \epsilon_p + \sigma B$.

The transverse dynamical susceptibilities χ^{+-} and χ^{-+} contain explicitly the magnetic field through ω^σ in the denominator of the expressions in the integral (in contrast to the longitudinal function χ^{zz}). As will be shown below, only this explicit field dependence is important for the appearance of the new contributions to $\chi(T)$ in the finite magnetic fields, and therefore we restrict ourselves to considering the transverse SF only from now on.

The following expression for evaluating the magnetic susceptibility will be used:

$$\frac{1}{\chi} = \left(\frac{\partial H}{\partial \boldsymbol{M}}\right) = \left(\frac{\partial^2 F}{\partial \boldsymbol{m}^2}\right) \tag{5}$$

whence, assuming δF to be a small correction, we get for the contribution of the transverse SF to the susceptibility

$$\chi(T) = \chi(0)\left(1 - \chi(0) \frac{1}{V} \frac{\partial^2 \delta F^{tr}}{\partial \boldsymbol{m}^2}\right) = \chi(0)(1 - \chi(0)\delta(T)) \tag{6}$$

where the second term is defined by the following expression:

$$\delta(T) = \frac{1}{V} \frac{\partial^2 \delta F^{tr}}{\partial \boldsymbol{m}^2} = -\frac{8}{\pi \chi_p} \frac{\partial^2}{\partial B^2} \sum_\sigma \int_0^\infty \mathrm{d}\omega\; N_\omega \int_0^\infty \mathrm{d}k\; k^2 \arctan\left(\frac{\mathrm{Im}\, \chi^{\sigma,-\sigma}}{-1/F_0^a - \mathrm{Re}\, \chi^{\sigma,-\sigma}}\right). \tag{7}$$

Here $F_0^a = \eta(0)\Psi_0$ ($\eta(0)$ is the density of states at the Fermi level), N_ω is the Planck function, and summation over σ takes into account only transverse SF. The term which is proportional to $\sum \chi^{\sigma,-\sigma}$ gives only an insignificant contribution to the temperature behaviour of the susceptibility and thus is excluded from further consideration.

The expressions (5)–(7) allow us to calculate the SF contribution to the susceptibility (see [5, 9, 12]).

3. The temperature dependence of $\chi(T)$ due to the SF

For the strongly enhanced paramagnets ($S \gg 1$) the main contribution to the integrals over k and ω is given by the long-wavelength and low-frequency SF, and this permits us to expand the dynamical susceptibilities in a series of powers of $s = \omega/4k \ll 1$, $s^\sigma = \omega^\sigma/4k \ll 1$, $k \ll 1$, and to keep only a finite number of terms in this expansion. For $\chi^{\sigma,-\sigma}(k, \omega, B)$ one can obtain [12]

$$\chi^{\sigma,-\sigma}(k, \omega, B) = 1 - ss^\sigma - \frac{1}{3}s(s^\sigma)^3 - \frac{1}{3}k^2 + \cdots + \mathrm{i}\frac{\pi}{2}s\Theta\left(k - \frac{|\omega^\sigma|}{4}\right)\Theta(1-k) \tag{8}$$

where the expansion coefficients correspond to a parabolic form of the quasiparticle spectrum.

Substitution of (8) into (7) gives

$$\delta(T) = -\frac{8}{\pi\chi_p}\frac{\partial^2}{\partial B^2}\sum_\sigma \int_0^\infty \mathrm{d}\omega\ N_\omega \int_{|\omega^\sigma|/4}^1 \mathrm{d}k\ k^2 \left[\frac{\pi}{2}\ \mathrm{sgn}\ \Phi_\sigma(k, \omega) - \arctan \Phi_\sigma(k, \omega)\right]. \tag{9}$$

Here we use the notation

$$\Phi^\sigma(k, \omega) = \frac{k}{\beta\omega} + \frac{1}{2\pi}\frac{\omega^\sigma}{k}\left[1 + \frac{1}{3}\left(\frac{\omega^\sigma}{4k}\right)^2\right] \qquad \beta = -\left(\frac{\pi}{8}\right)\left(\frac{F_0^a}{1+F_0^a}\right)$$

and the relation: $\arctan x = (\pi/2)\mathrm{sgn}(x) - \arctan(1/x)$.

The integration over k in (9) can be performed explicitly. However, in the general case the result will have a complicated form and so we do not write it out here. Let us remark only that the character of the dependence of this expression on ω and B will be determined by the parameter $\omega^\sigma/(\beta\omega)$. The new contributions which we are interested in arise only at small values of this parameter—that is, for $\omega^\sigma/(\beta\omega) \ll 1$, or, if one notes that the main contribution to the integral over ω arises from the region $\omega \sim T$, for $T \gg H$.

These new contributions are proportional to $(\omega|\omega^\sigma|)^{3/2}\Theta(2B - \omega)$ (this term originates from the first term in the square brackets in (9)) and to $(\omega^\sigma)^3 \ln|\omega^\sigma|/\omega$.

For an illustration of these statements, we describe the simplified calculation of these terms, based on expansion of the arctangent as a series in its argument. Here one must take into account the possibility of a changing of the character of the arctangent's expansion at the point k^*, where the argument is equal to 1 (in our case k^* approximately equals $\beta\omega$). Thus, the condition $\omega^\sigma/(\beta\omega) < 1$ for the appearance of the new contributions corresponds to the case where k^* falls within the range of integration over k.

We can therefore split the region of integration over k into $|\omega^\sigma|/4 < k < \beta\omega$ and $\beta\omega < k < 1$, and consider further only the first interval (where arctangent's argument is less than 1). Thus, the fluctuation factor (9) may be written in the form

$$\delta(T) = -\frac{8}{\pi\chi_p}\frac{\partial^2}{\partial B^2}\sum_\sigma \int_0^\infty \mathrm{d}\omega\ N_\omega \int_{|\omega^\sigma|/4}^{\beta\omega} \mathrm{d}k\ k^2 \left\{\frac{\pi}{2}\ \mathrm{sgn}\left(\frac{k}{\beta\omega} + \frac{2}{\pi}s^\sigma\right) - \frac{k}{\beta\omega} - \frac{2}{3\pi}\left(1 - \frac{4}{\pi^2}\right)(s^\sigma)^3\right\} \tag{10}$$

where only the terms responsible for the essential contributions to the temperature dependence of the susceptibility are retained.

Let us now consider the integrals over k from each term in the curly brackets in (10).

(i) The main contribution to the integral

$$I_1 = \frac{\pi}{2}\int_{|\omega^\sigma|/4}^{\beta\omega} \mathrm{d}k\ k^2\,\mathrm{sgn}\left(\frac{k}{\beta\omega} + \frac{2}{\pi}s^\sigma\right) \tag{11}$$

arises at $k \sim k_0 = \sqrt{|\omega^\sigma|\beta\omega/(2\pi)}$. Taking this into account, we can obtain for I_1

$$I_1 = -\frac{\pi}{3}\left(\frac{\beta\omega}{2\pi}|\omega^\sigma|\right)^{3/2}\Theta(2B - \omega). \tag{12}$$

(ii) The result of integration over k of the second term in (10) is

$$I_2 = -\frac{1}{\beta\omega}\int_{|\omega^\sigma|/4} \mathrm{d}k\ k^3 = \frac{(\omega^\sigma)^4}{4^5\beta\omega}. \tag{13}$$

Evaluating the contribution from I_2 to $\chi(T)$ one can conclude that no significant temperature corrections occur.

(iii) The third integral contains the terms which are non-analytical when $H \to 0$:

$$I_3^\sigma = -\frac{2}{3\pi}\left(1 - \frac{4}{\pi^2}\right)\int_{|\omega^\sigma|/4}^{\beta\omega} \mathrm{d}k\ k^2 (s^\sigma)^3 = \frac{2}{3\pi}\left(1 - \frac{4}{\pi^2}\right)\left(\frac{\omega^\sigma}{4}\right)^3 \ln\frac{|\omega^\sigma|}{4\beta\omega}. \tag{14}$$

After integrating over ω this expression results in the logarithmic term in the susceptibility.

If the condition $H \ll T$ is satisfied, the value of the integral I_2 is much smaller than those of I_1 and I_3, and, therefore, for $\delta(T)$ we can write

$$\delta(T) = -\frac{8}{\pi\chi_p}\frac{\partial^2}{\partial B^2}\int_0^\infty \mathrm{d}\omega\ N_\omega\left(I_1 + \sum_\sigma I_3^\sigma\right). \tag{15}$$

Differentiating twice with respect to B, and carrying out the simple transformations with account taken of (12) and (14), we get

$$\delta(T) = \frac{1}{\chi_p}T^2\left\{\frac{1}{8}S^{3/2}J_1(\ell) - \frac{2}{\pi^4}\left(1 - \frac{4}{\pi^2}\right)(J_2(\ell) - \ell J_3(\ell))\right\} \tag{16}$$

where

$$J_1(\ell) = \int_0^\ell \frac{\mathrm{d}z}{\mathrm{e}^z - 1}\ \frac{z}{(\ell/z - 1)^{1/2}} \tag{17}$$

$$J_2(\ell) = \int_0^\infty \frac{\mathrm{d}z}{\mathrm{e}^z - 1}\ z\ln\left|\frac{1 - (\ell/z)^2}{\beta^2}\right| \tag{18}$$

$$J_3(\ell) = \int_0^\infty \frac{\mathrm{d}z}{\mathrm{e}^z - 1}\ \ln\left|\frac{z + \ell}{z - \ell}\right|. \tag{19}$$

Thus, one can see that $\delta(T)$ really contains non-analytical (with respect to ℓ) terms. We note once again that these results (which lead to the new contributions to the susceptibility) could be obtained without making use of the approximate arctangent expansion, just by performing the explicit integration over k in the initial expression (9) (using the expansion (8) for $\chi^{\sigma,-\sigma}(k,\omega,B)$ and the condition $|\omega^\sigma|/4 \ll k^* \ll 1$).

Now the behaviour of $\delta(T)$ and $\chi(T)$ must be analysed in the extreme cases where the closed form for the new SF terms in the susceptibility may be derived.

(i) $\ell \ll 1$ (the extremely weak magnetic fields).

The approximate expressions for integrals $J_1(\ell)$ and $J_2(\ell)$ have the forms

$$J_1(\ell) = \int_0^{\ell} \frac{\mathrm{d}z}{(\ell/z - 1)^{1/2}} = \frac{\pi}{2}\ell \tag{20}$$

$$J_2(\ell) = \int_0^{\infty} \mathrm{d}z \; \frac{z}{\mathrm{e}^z - 1} \ln\frac{1}{\beta^2} = -\frac{\pi^2}{3}\ln\beta. \tag{21}$$

The numerical analysis of the integral $J_3(\ell)$ versus ℓ shows that at small ℓ ($\ell \sim 10^{-3}$–10^{-4}) $J_3(\ell)$ behaves as a smooth function of ℓ, being approximately equal to a constant: $J_3(\ell) \approx \text{constant} = A$ ($\sim$4.7).

Then for $\delta(T)$ we get

$$\delta(T) = \frac{1}{\chi_p}\left[\left(\frac{\pi}{8}S^{3/2} + \frac{4A}{\pi^4}\left(1 - \frac{4}{\pi^2}\right)\right)BT + \frac{2}{3\pi^2}\left(1 - \frac{4}{\pi^2}\right)T^2\ln\beta\right]. \tag{22}$$

Substituting (22) into the expression for the susceptibility, equation (10), we obtain a result which partially reproduces the contribution obtained in previous work [5] (it reproduces it totally if the longitudinal SF is taken into account and the quantum approach rather than the quasiclassical one is used). Thus, it is not the case that we are interested in here.

(ii) $\ell \gg 1$ (the finite fields).

For the integrals J_1–J_3 the following approximate expressions are valid:

$$J_1(\ell) = \int_0^{\infty} \frac{\mathrm{d}z}{\mathrm{e}^z - 1}\,\frac{z^{3/2}}{\ell^{1/2}} = \frac{3}{4}\sqrt{\frac{\pi}{2}}\,\zeta(5/2)\left(\frac{T}{B}\right)^{1/2} \tag{23}$$

$$J_2(\ell) = 2\int_0^{\infty} \mathrm{d}z \; \frac{z}{\mathrm{e}^z - 1}\ln\frac{\ell}{\beta z} = -\frac{\pi^2}{3}\ln\frac{T}{T^*} \tag{24}$$

$$J_3(\ell) = \frac{2}{\ell}\int_0^{\infty} \mathrm{d}z \; \frac{z}{\mathrm{e}^z - 1} = \frac{\pi^2}{6}\,\frac{T}{B} \tag{25}$$

where $\zeta(x)$ is the Riemann zeta function, and T^* is a characteristic temperature ($T^* \sim B$). So, for $\delta(T)$ we have

$$\delta(T) = \frac{1}{\chi_p}\left[\frac{3}{32}\sqrt{\frac{\pi}{2}}\,\zeta(5/2)S^{3/2}\frac{T^{5/2}}{B^{1/2}} + \frac{2}{3\pi^2}\left(1 - \frac{4}{\pi^2}\right)ST^2\ln\frac{T}{T^*}\right]. \tag{26}$$

Therefore, the temperature expansion of the susceptibility for $\ell \gg 1$ has the form

$$\chi(T) = \chi(0)\left[1 - \frac{3}{32}\sqrt{\frac{\pi}{2}}\,\zeta(5/2)S^{5/2}\frac{T^{5/2}}{B^{1/2}} - \frac{2}{3\pi^2}\left(1 - \frac{4}{\pi^2}\right)ST^2\ln\frac{T}{T^*}\right]. \tag{27}$$

Here the T^2-term from $J_3(\ell)$ was introduced within the logarithm renormalizing the temperature T^*. It should be noted that the S^2T^2-contribution to $\chi(T)$ obtained by Beal-Monod *et al* [5] is really present in the temperature expansion both for $\ell \ll 1$ and for $\ell \gg 1$, but we have no need to reproduce this term in our calculations (as already mentioned, to obtain it we must take into consideration the longitudinal SF and finite k). This T^2-term will be written down in the final expressions only. We underline also that our $ST^2\ln T$-term ($\ell \gg 1$) originates from the same free-energy terms as produce the $ST^2\ln S$-term obtained by Beal-Monod *et al* [5] for $B = 0$.

At the end of this section we additionally emphasize that these new contributions were not revealed in the previous works since there only the case of extremely weak magnetic field ($\ell \ll 1$) was considered.

4. Discussion

We have established that the temperature dependence of the susceptibility of exchange-enhanced paramagnets $\chi(T)$ has the form

$$\frac{\chi(T)}{\chi(0)} = \begin{cases} 1 - \alpha S^2 T^2 & \text{when } T \ll H \text{ or } T \gg 2SH \\ 1 - \alpha S^2 T^2 - \delta S^2 \dfrac{T^{5/2}}{H^{1/2}} - \gamma S T^2 \ln \dfrac{T}{T^*} & \text{when } H \ll T \ll 2SH \end{cases} \tag{28}$$

where $\alpha = 7.7\pi^2/24$ [5, 6], $\delta = (3\sqrt{\pi}/64)\zeta(5/2)$, and $\gamma = (2/3\pi^2)(1 - 4/\pi^2)$.

So, the consideration of the case of the finite magnetic field has resulted in the appearance of the 'singly enhanced' logarithmic contribution to $\chi(T)$, and it is in agreement with a qualitative result of Misawa [4] obtained with the help of the phenomenological Fermi-liquid approach. Moreover, this logarithmic contribution exists only at finite temperatures ($T \gg H$), which make it possible to avoid the problem indicated by Beal-Monod [11] (the singularity of $\gamma(H) = C_v(H)/T$ as $T \to 0$).

Another important result is that the logarithmic term is accompanied with a more significant contribution, proportional to $T^{5/2}/H^{1/2}$. The coefficient of this term is proportional to $S^{5/2}$ (while the coefficient of the logarithmic term is $\sim S$), and this is why the logarithmic term can become comparable with the $T^{5/2}$-term only in a case where the temperature T^* is extremely high ($T^* \sim \exp S,\ \ S \gg 1$), but this seems to be improbable. This means that for the range of temperatures $H \ll T \ll 2SH$ the $T^{5/2}/H^{1/2}$-term plays the main role. The sign of this term is the same as that of the usual T^2-term due to the SF, and this means that this term cannot be responsible for the non-monotonic behaviour of the susceptibility. Nevertheless, more realistic parametrization of the Landau function (e.g., keeping at least two Fermi-liquid constants) can result in a change in sign of the $T^{5/2}/H^{1/2}$-term. For instance, for liquid ^{3}He the making use of the second Fermi-liquid parameter F_a^1 significantly changes the magnitude of the coefficient of the logarithmic term in the SF specific heat and allows one to explain its temperature behaviour quantitatively [13].

But even in the present model this term can significantly change the character of the temperature dependence of $\chi(T)$. Writing out the expression for the susceptibility for the range $H \ll T \ll 2SH$ in a form convenient for comparison with experimental data, we get

$$\frac{\chi(T) - \chi(0)}{\chi(0)} = -b[\chi(0)T]^2 \tag{29}$$

where

$$b = -\frac{4}{9}\alpha \left\{ 1 + \frac{\delta}{\alpha} S^{1/2} \sqrt{\frac{2T}{B}} \right\}. \tag{30}$$

This leads us to the conclusion that at finite fields ($\ell \gg 1$) and for strong paramagnets ($S \gg 1$) the new contribution appreciably alters the magnitude of b as well as giving rise to the strong temperature and field dependence of this factor, whereas the results of previous works on the SF T^2-term imply that b has only weak temperature (due to the terms of the next order in T^2) or field dependence. So, it may become necessary for strong paramagnets to take into account this dependence for the correct interpretation of the experimental data on the temperature dependence of $\chi(T)$ (and this is the subject of another study based on re-examining numerous existing experimental data for different compounds to extract evidence for or against this interpretation).

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